Executive Summary

This project explores the interplay between target-date funds and annuitization. Target-date funds, which represent age-dependent combinations of equity and bonds, span the asset allocation space without annuitized investing. The “target-date funds” approach highlights the dependence of the optimal asset allocation between risky and riskless assets on the investor’s age. Furthermore, if the Capital Asset Pricing Model (CAPM) holds, then all target-date funds are on the mean-variance efficient frontier. Target-date funds can be introduced into annuitized investing. An individual or couple can own variable annuities whose notional value adjusts with market movements so the insured entity can hedge financial market risk while fully insuring idiosyncratic mortality risk. Financial/portfolio risks and mortality risks are independent and separable, leading to a “separation theorem” so that the structure of the holdings (including annuities) can capture completely both market risks and idiosyncratic mortality risk—rather than distorting the asset allocation in order to address more fully idiosyncratic mortality risk. In contrast, the “target-date funds” framework without annuities would not allow an individual with a limited bequest motive to maximize expected utility because of an inability to insure mortality risk. Indeed, the spanning and separation results can be interpreted as supporting the use of target-date fund products within annuitized vehicles. The optimal location of annuitized investing also is explored and asset location (where to locate equity and bonds) within such contexts is examined.
1. Introduction

Target-date funds in which the asset allocation changes over time based upon the investor’s age or projected retirement date have played an increasingly important role in individual investing, especially in the context of retirement plans. While these target-date funds represent a combination of equity and fixed-income investments (or risky and riskless investments), such funds play an important foundational role in asset allocation. The traditional view has been to suggest that the target-date fund for the investor’s age should guide his investment, while recognizing that the investor has a range of alternative allocations that he could wish to select. Instead of viewing investment through the target-date fund for the investor’s specific age as being narrowly prescriptive and determined by theory, we reverse direction and allow the investor flexibility in selecting the specific target-date funds and examine the foundational properties of these products.

We assume that the proportion of equity in the target-date fund decreases with the investor’s age. Since target-date funds represent a unique linear combination of equity and bonds, we observe that two target-date funds can be used to replicate all other target-date funds and span all equity-bond mixes. These target-date funds are on the mean-variance frontier since they allow the investor to replicate any combination of the market basket and the riskless asset. In effect, the Capital Asset Pricing Model (CAPM) is equivalent to the efficiency of target-date funds. These findings are consistent with the use of target-date funds within annuitized products; the target-date fund is used to achieve the desired financial risk exposure, while the annuitization structure insures the individual’s mortality at the desired level.

In Section 2 we discuss the properties of target-date funds, including how they span the investment space and the efficiency of target-date funds under the Capital Asset Pricing Model. We extend our results on target-date funds to the presence of annuities in Section 3. We consider the impact of taxes on asset location with annuitization in Section 4 and conclude in Section 5.

2. Target-date funds and efficient investing

Much of the emphasis in recent years on investment of retirement funds has focused upon “target-date funds,” which entail investing in a combination of mutual funds in which the investor’s underlying asset allocation between equity and fixed-income is a function of an investor’s age. The investor’s choice among such target-date funds can reflect a desired asset allocation rather than the investor’s actual age. In effect, the target-date fund can correspond to that for someone who has a different attained age. This, itself, can be adjusted over time. A target-date fund is a portfolio mix determined by a hypothetical age for the investor, which is specified by mutual fund organizations for direct investment or selected by employers as alternatives within the context of retirement accounts. Typically, the target-date funds are structured so that the proportion of equity declines with the investor’s age.1

Assumption A: The proportion of the target-date fund invested in equity strictly declines with the investor’s age.

This assumption, which is satisfied (in weak form) for target-date fund programs in practice, can be motivated in several ways. In a typical program there is a discrete set of target-date fund options, so that the equity proportion can be cast as a non-increasing step function. However, by taking a convex combination of target-date funds at nearby ages, the equity proportion strictly declines over a continuous set of ages. For example, the shortening horizon of an investor as he ages and declining remaining longevity or life expectancy suggest declining risk-taking as the investor ages. Furthermore, there are important links to human capital over the life cycle. Human capital represents part of the individual’s total wealth, but it is not included in standard measures of financial wealth. As the investor ages, his remaining human capital wealth declines (both in absolute terms and relative to both his financial wealth and total wealth) due to more limited opportunities to utilize his skills. This points to several additional motives for the investor’s allocation of his financial wealth to risky assets (e.g., equity) to decline as he ages in that (a) the investor’s financial wealth is an increasing proportion of his overall

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1. This is an aspect of how target-date funds are described and structured. A broader discussion of target-date funds is provided by Mitchell and Utkus (2012).
wealth (younger people may want to invest financial wealth aggressively because it is such a small proportion of their overall wealth) and (b) human capital tends to be viewed as less risky than financial wealth (it is especially less risky than equity), so it becomes relatively less important as the investor ages and so the allocation of financial wealth should become more cautious. Of course, the human capital effects alone would not be significant once the individual has fully retired, though the shortening horizon would be particularly significant then. Ultimately, the investor selects the asset allocation that optimizes his objective among the alternatives. Distinct portfolio mixes in target-date funds are typically offered for several distinct points on the age spectrum, but the investor is free to select any of these target-date funds or age-dependent portfolio mixes (rather than the specific one for his age). The approach here is to highlight the nature and set of target-date fund investments rather than suggesting that the target-date fund for a particular age should be selected by an investor of that age.

While, in practice, mutual fund organizations offer a limited set of target-date funds, conceptually there is a dense set of such target-date funds (each corresponding to a unique proportion of equity and risk-free assets). In effect, the choice among target-date funds by the investor is equivalent to selecting an asset allocation. This can be summarized by the conclusion that two target-date funds composed of mixes of equity and risk-free bonds fully span the feasible asset allocations. Also, we note that two target-date funds held in appropriate proportions replicate any other target-date fund.

**Proposition 1**

A linear combination of two target-date funds spans the feasible allocations of bonds and equity and replicates any other target-date fund. In effect, this result provides a form of two-fund separation within the subspace of target-date funds. Here, the separation idea emerges as a direct byproduct of the definition of target-date funds as a combination of equity and fixed-income portfolios. So far, we have not argued that target-date funds are optimal investments. However, taking the conclusion in Proposition 1 together with underlying restrictions on preferences or the distribution of returns leads to the optimality of such portfolio combinations, as in the classic papers on two-fund separation. In a frictionless setting and in the presence of a risk-free asset, sufficient conditions for two-fund separation imply that the optimal composition of the risky portfolio is identical for all investors.

Another form of the “separation theorem” is that any efficient portfolio can be represented by a target-date fund, which we interpret as a combination of the risky and riskless assets, i.e., the market portfolio and riskless assets. Since the standard formulation of the Capital Asset Pricing Model (CAPM) implies that the CAPM is equivalent to the efficiency of the market portfolio, the CAPM also is equivalent to any target-date fund being an efficient (mean-variance frontier) portfolio under our interpretation of the target-date fund.

**Proposition 2**

The CAPM is equivalent to the mean-variance efficiency of any target-date fund.

This offers a fundamental rationale for target-date funds, while not recommending particular target-date funds as expected utility maximizing to specific investors. At the same time our analysis does not evaluate which investors should select which specific products.

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2. Some observers interpret the compensation of tenured faculty as somewhat like a bond (relatively low risk) compared to other occupations. More broadly this highlights the possibility of differences in “financial/market” risk among occupations, which would influence the choice of asset allocation.

3. This is analogous to the idea that call (or equivalently put) options span the feasible payoffs. For example, every pure contingent claim (e.g., Arrow-Debreu security) that pays off in a single state can be replicated by a portfolio of options. Consequently, options can replicate all potential contingent claims (see the classic papers by Ross (1976) and Breeden and Litzenberger (1978)).

4. This assumes the possibility that one of the target-date funds could be sold short, i.e., the fund being replicated need not be a convex combination of the two fixed funds, though they are a linear combination of these.

5. Two-fund separation would emerge under such assumptions as quadratic utility for preferences or normally distributed one-period returns (see Tobin (1958)). The classic papers on two-fund separation more broadly include Cass and Stiglitz (1970) and Ross (1978) as well as Tobin (1958).

6. Two-fund separation applies directly to after-tax payoffs in the presence of taxes under the assumption that all investors are taxed identically. The issue becomes more delicate in the presence of heterogeneous taxation across investors or multiple tax treatments (such as taxable and tax-deferred funds) for a given investor. Indeed, in the presence of diverse (constant) tax rates there does not exist equilibrium in a tax clientele setting.
3. Annuitization and target-date funds

An important asset category that is often not emphasized in traditional discussions of asset allocation (for “target-date funds” and even otherwise) is annuities. Annuities represent a type of asset defined on a different dimension than equity and bonds—annuities insure longevity. In its simplest form an annuity makes level payments throughout the life of the insured and then does not make further payments. In contrast, traditional asset allocation reflects the choice of financial assets whose payoffs are defined in terms of the state of the economy (or firm) rather than the specific mortality status of an individual (or a couple). However, one can hold risky assets and riskless assets outside or inside an “annuity wrapper.” In the latter instance the payoff would be determined by the state of the economy as well as whether the individual is alive. The “annuitization puzzle” represents a critique of the failure of many investors to hold annuities, even when they lack a strong bequest motive. Indeed, this critique is that many individual investors do not annuitize at all, rather than that they do not fully annuitize. Of course, the integration of annuities into the portfolio setting provides a mechanism to obtain additional consumption than can be achieved by investors owning only assets that do not provide insurance against longevity. By annuitizing and insuring his longevity, the consumer-investor obtains a larger return that fully utilizes his resources during his lifetime. In contrast, without annuitization the consumer would obtain a lower effective return and die with assets in his name. The insurance provided through annuitization allows such assets to be consumed during the investor’s lifetime, rather than retaining resources at death that can be used to fulfill a bequest motive.

In contrast to the case with annuities, the traditional “target-date funds” framework without annuities does not allow investors without a bequest motive or only a modest bequest motive to maximize their expected utility. By incorporating annuities into the “target-date funds” framework we would facilitate utility maximization. Interpretation of the optimal solution in the presence of annuities would provide basic insight about how “target-date funds” could work with annuities.

In a standard asset allocation framework the employee’s optimal asset allocation involves holdings of risky (market) and riskless assets. The “target-date funds” approach highlights the dependence of this asset allocation upon the investor’s age. Of course, individuals face not only market risk that could be hedged through ownership of risky securities, but they also face idiosyncratic mortality risk that can be hedged through annuities.

One can shift value among the assets in the annuity wrapper each period; the annuity structure can potentially account for both a fixed and variable annuity and allow redistribution of value between the two assets. The initial purchase of the annuity or deferred annuity can be a blend of a fixed and variable annuity—the mortality risk reflects the market value of the combined fixed and variable annuity. We assume that there is no correlation between the equity return and mortality realizations for the individual (the individual is small compared to the market; furthermore except in extreme instances individual mortality would not influence the equity return, and market returns do not influence individual or even aggregate mortality).

Assumption B: The realization of market risk and mortality risk are assumed to be independent.

For the most part this is a very reasonable assumption, as illustrated by the types of instances in which the assumption that individual mortality is independent of market returns would not be satisfied. For example, one could imagine that the death of a President of the United States or an extraordinary innovator (e.g., Steven Jobs) would be correlated with and influence market returns. Certainly at the level of an individual firm there could be such impacts (we’ve even seen positive responses to rumors of the death of underperforming CEOs).

The annuitization puzzle was highlighted by Modigliani (1986) and more recently surveyed by Benartzi, Previtero and Thaler (2011). Milevsky and Young (2007) highlight that adverse selection can help explain the delay in purchasing annuities.

Individuals could be reluctant to fully annuitize because of a limited bequest motive or because of the desire to accommodate lumpy consumption demands and needs. A weakness of the interpretation that individuals do not partially annuitize (in the United States) is that Social Security (which is essentially universal) actually provides a real annuity.

This would be clearly inefficient absent a bequest motive.

It is also possible that the causal relation between market risk and mortality risk be reversed, e.g., market fluctuations could lead to financial or employment uncertainty and subsequent detrimental impacts on health.
Under these assumptions one can reallocate value each period between the assets within the annuity wrapper. Because the individual can own variable annuities whose notional value adjusts with market movements (along with fixed annuities, whose value would be invested in bonds), the individual can fully insure the relevant risks—both market risk and the idiosyncratic mortality risk to the extent that he desires to do so. In effect, there is an underlying “separation theorem” such that the structure of assets (including the various annuities) can capture fully both market risk and idiosyncratic mortality risk once annuities are considered. The import of this “separation theorem” is to demonstrate that the employee’s optimal exposure to market risk (taking into account the implicit equity in any variable annuities that he holds) would be identical to his optimal holding of market risk absent the availability of annuities. In effect, the optimal annuitization decision and the asset allocation are separable.

Proposition 3
The optimal annuity purchase decision is separable from the optimal asset allocation under the assumption of independent financial returns and mortality realizations. Hence, the target-date funds offered inside the annuity wrapper should mirror those offered outside the annuity.\textsuperscript{11}

One of the most significant frictions that influences annuitization would be the adverse selection costs associated with annuity purchases (annuities would be especially likely to be purchased by those with considerable anticipated longevity, thereby limiting the use of annuities somewhat by reducing the returns offered in the marketplace and highlighting their lack of attractiveness to those with especially high or perhaps even average mortality). The nature of adverse selection for annuities is the opposite of that for life insurance. The individuals with high mortality are especially interested in purchasing life insurance, while for annuities the selection bias is to encourage annuitization for those with likely longevity. Despite the adverse selection, for many individuals, greater emphasis on annuities would be attractive, though the adverse selection can discourage or delay the purchase of an annuity (see Milevsky and Young (2007)).

4. Taxes and annuitization

A second important friction would be to adjust for differential tax treatment between taxable and tax-deferred funds. As highlighted in Dammon, Spatt and Zhang (2004), the division of an investor’s wealth between taxable and tax-deferred funds plays a crucial role with respect to asset location and allocation. For example, the optimal extent of holdings of equity is influenced by whether the marginal location for equity would be in the taxable or tax-deferred account. Similar logic will help pin down the indirect impact of taxes upon annuity holdings in a target-date funds context. In the traditional context with taxable and tax-deferred investing, Dammon, Spatt and Zhang (2004) point out that the taxation of bond returns and equity returns is neutral within the tax-deferred account, while equity has a considerable tax preference within a standard taxable account.\textsuperscript{12} This leads to their asset location conclusion that one should place his desired fixed-income exposure first in the tax-deferred account and the desired equity first in the taxable account. The overall desired risk exposure interacts with the split of wealth in determining the asset location mix.

In contrast, equity and bonds are taxed identically within an annuitization context, whether in a tax-sheltered retirement setting or taxable context. This has implications for both the placement of equity vs. bonds within annuities compared to non-annuity contexts and the location of exposures across the various buckets. Then, the asset location (equity vs. bonds and specific target-date funds) is crucially linked to the allocation of wealth among the various contexts. The neutral treatment of bonds and equity, in both tax-sheltered settings, generally, and annuities, points to non-annuitized, taxable funds being the first place to locate equity. The potential role of equity in annuities with taxable funds (and whether they have a greater or lesser potential role than in annuities in retirement accounts) is ambiguous. On the one hand, the equity is subject to tax deferral until funds are withdrawn from the annuity for consumption (with an adjustment for principal).\textsuperscript{13} On the other hand, the preferential tax rate on equity returns is eliminated.

\textsuperscript{11} This conclusion may be sensitive to the absence of adverse selection about mortality risk.

\textsuperscript{12} Favorable tax rates are applied to both dividends and capital gains and additionally, the taxation of capital gains is deferred and the timing of realization is endogenous. The endogenous deferral of capital gains in a setting in which taxes are deferred until realization (but not in an annuity structure) is undertaken by Balcer and Judd (1987).

\textsuperscript{13} More detailed discussion and analysis of the tax treatment of the withdrawals from annuities (including allocation of payments to principal) is undertaken by Brown, Mitchell, Poterba and Warshawsky (1999) and Warshawsky (2015).
Indeed, in the current environment (low time value, but a large differential between equity and bond taxation) we would anticipate that the value of the deferral is not as substantial as the large tax rate differential being foregone with equity investments.\(^{14}\)

The allocation of wealth to annuities is determined by the individual. One point to emphasize is that investing in a tax-deferred context (with or without annuitizing) dominates from a tax perspective of investing in annuities in a taxable context. This is because the former essentially does not tax the growth of the funds, while the latter subjects the growth to some taxation at the withdrawal of the annuity payments.\(^{15}\) Hence, from a tax perspective, annuitizing would be recommended in a taxable setting only when the limit on contributions to the tax-deferred account binds.

Of course, most individuals in the United States have some degree of annuitization within their retirement funding, i.e., Social Security. This is likely to have a substantial impact for those individuals whose bulk of retirement income is in Social Security (individuals with relatively low income). This suggests that the greatest potential for annuitization would be by relatively higher income individuals.

While much of the viewpoint sketched here is from the perspective of an employee, target-date funds are sometimes selected by employers (plan sponsors) or asset managers at a prior ex ante stage.\(^{16}\) To the extent that a dense/continuous set of target-date funds can be offered and the investor/beneficiary is not restricted to one based upon his own birthdate, the resulting heterogeneity in circumstances would not be restrictive. In that sense a dense set of target-date funds does not restrict the investor’s choice when the individual can select a target-date fund for any date.

5. Concluding comments

This paper provides perspective on the foundation of target-date fund investments both without and with the potential of the investor to annuitize. Target-date funds are convex combinations of equity and bond funds. Any target-date fund or asset allocation is a linear combination of risky equities and riskless bonds, which is spanned and can be replicated by two target-date funds. While investors won’t necessarily find it optimal to choose the target-date fund corresponding to their particular age (they can select any target-date fund), the CAPM is equivalent to the efficiency of target-date fund investments. Furthermore, the foundation and role of target-date funds to manage risks in one’s financial account is robust to the presence of the opportunity to annuitize and does not distort the annuitization decision.

\(^{14}\) The value of the deferral may be further enhanced because the ordinary income tax rate applied to the growth of the annuity (or to withdrawals from a tax-deferred account) would typically reflect the individual’s retirement income rather than his income from his working life.

\(^{15}\) Dammon, Spatt and Zhang (2004) discusses why the growth of the tax-deferred account is effectively not taxed (i.e., the original contribution was deductible, so the government has a claim on its portion of the account, reflected in the taxation of withdrawals). In contrast, the contribution to an annuity in a taxable account is not deductible, but the growth would be taxed at withdrawal (this is somewhat analogous to an “after-tax” IRA; on the other hand, a Roth IRA provides no deduction up front and no tax of the withdrawals).

\(^{16}\) Employers could design the plan based upon the ex ante distribution of investor preferences, recognizing that there is a spectrum of employee circumstances.
References


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